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Robust Controller Design for Non-Minimum Phase Zero Systems Using Algebraic Approach and Pole Placement method

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Abstract: The non minimum phase systems are difficult to control because of RHP zero. This paper the robust controllers are designed for non-minimum phase systems based on algebraic approach and pole placement approach. The controller coefficients are calculated from general solutions of Diophantine equations. The robustness of the controller are verified in simulation in the mat lab environment to show the effectiveness of these designed controllers rejecting disturbances and also in alternating the noise characteristics. Time domain and frequency domain simulations are performed on the non-minimum phase systems.

Keywords: Algebraic approach, robust control, Non-minimum phases zeros, pid controllers, Pole placement method.

I. INTRODUCTION

A PID controller is a widely used feedback controller in industrial process in the market since 1939 and has remained as the mostly used controller in process control until today. However, the choice of weighting the three individual actions i.e. proportional, integral and derivative has been a problem. In this paper, Algebraic approach is adopted from algebraic approach (4),(5) and (6) as an effective tool for control design. This technique is based on the general solutions of Diophantine equations. The main advantage of this approach is that, the behaviour of the controller can be tunned by only one tunning parameter m>0.
The other method used for the control design is the classical pole placement technique using polynomials. In this method the roots of the characteristics polynomial are placed in arbitrary locations.
A system that has none or asymptotically stables zero dynamics is called minimum phase. Otherwise the system is non-minimum phase .it is directly possible to conclude from the position of the invariant zeros to the stability of the zero dynamics.
The response of a non minimum phase system to a step input has an “undershoot”. This means, if the output was initially zero and steady state output is positive, the output becomes first negative before changing direction and converging to its positive steady state value. Both algebraic approach and pole placement methods are used to derive the controller for these non-minimum phase systems.

II. ALGEBRAIC CONTROL DESIGN IN R_{PS}

2.1 Transfer function in R_{PS}

The RPS is the ring of proper and Hurwitz-stable rational functions. The properness of function means that the degree of polynomial in its denominator is higher or at least equal as the degree of polynomial in its numerator. The stability is ensured by location of all poles in left complex half plane.

$$\frac{s}{s+1};\frac{s-1}{(s+3)^2};\frac{3}{s+5}\in R_{PS}\,;\quad s;\frac{1}{s};\frac{s+2}{(s+3)^3}\notin R_{PS} \tag{1}$$

The conversion of the polynomial representation to the RPS notation is simple. It is just the division of both numerator and denominator by the same stable polynomial of appropriate order. Therefore the transposition can take a form,

$$G(s)=\frac{b(s)}{a(s)}=\frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}}=\frac{B(S)}{A(S)} \tag{2}$$

Where $m > 0$ is a free parameter and $n = \max\{\deg a(s), \deg b(s)\}$. The choice of multiple root m >0 brings into the synthesis the single real scalar tuning parameter which will be used as a tool influencing the properties of closed-loop control responses.

2.2 Controller design

This method supposes description of linear systems in RPS bounded with classical transfer Function by relation (2). The parameter $m > 0$ can be used as a “tuning knob” for influencing of final control response.
The general closed control loop with presence of disturbance signals can be realized according to fig.1. It should be noted that all functions and signals represented in fig.1.are considered to belong to RPS.

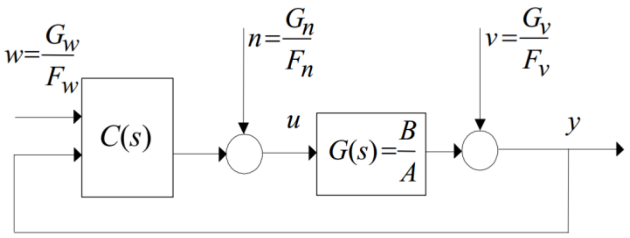

Fig.1. General control loop

This circuit can have separated feedback $C_b(s)=Q_c(s)/P_c(s)$ and feed forward $C_f(s)=R_c(s)/ P_c(s)$part (control system with two degrees of freedom – 2DOF, FBFW).
In that event, assuming zero disturbances (n = v = 0), control signal u is generated by:

$$u = (C_f \; C_b)\,(\frac{w}{-y}) = C_f w - C_b y \tag{3}$$

Signals w, n, v represent reference value, load disturbance in the input and disturbance in the output of the controlled plant, respectively. Usually, w and n are considered as step signal and disturbance v is modelled to have a harmonic shape. Hence, the denominators of these signals in R_{PS} are;

$$F_w = F_n = \frac{s}{s+m}\,;\, F_v = \frac{s^2+\omega^2}{(s+m)^2} \tag{4}$$

Where ω is angular frequency and $m > 0$
The first and the most important requirement is to ensure the stability of control loop from fig.1. Stabilizing controllers are given by ratio:

$$\frac{Q_C}{P_C} = \frac{Q_C - AF}{P_{CO} + BF} \tag{5}$$

Where F is free in R_{PS}, $P_{CO} + BF \neq 1$, Q_{CO}, P_{CO} is some particular solution of Diophantine equation:

$$AP_C + BQ_C = 1 \tag{6}$$

2.3 Derivation of controller for Second order system

Mathematical model

The whole process of controller design, described above, can be illustrated by representative simple synthesis for second order controlled plant.

$$G(S) = \frac{b_0 - b_1 s}{s^2 + a_1 s + a_0} = \frac{\dfrac{b_0 - b_1(s)}{(s+m)^2}}{\dfrac{s^2 + a_1 s + a_0}{(s+m)^2}} = \frac{B(S)}{A(S)} \tag{7}$$

After transposition of all transfer functions in RPS the basic "stabilizing" Diophantine equation (6) can be written in the form.

$$\frac{s^2+a_1s+a_0}{(s+m)^2}\frac{p_1s+p_0}{(s+m)}+\frac{b_0-b_1s}{(s+m)^2}\frac{q_1s+q_0}{s+m}=1 \qquad \textbf{(8)}$$

After comparing left and right hand side terms of (7) the general solution of (6) can be expressed as,

$$P=\frac{s+p_0}{s+m}+\frac{b_0-b_1s}{(s+m)^2}T$$
$$Q=\frac{q_1s+q_0}{(s+m)}-\frac{s^2+a_1s+a_0}{(s+m)^2}T \qquad \textbf{(9)}$$

Where

$$d=3m^2-a_0-3a_1m+a_1^2+\frac{b_1}{b_0}m^3-\frac{b_1}{b_0}a_0(3m+a_1)$$
$$q_1=\frac{d}{b_0+\frac{b_1^2}{b_0}a_0+a_1b_1} \qquad \textbf{(10)}$$
$$p_0=3m-a_1+b_1q_1$$
$$q_0=\frac{m^3-a_0p_0}{b_0}$$

Were obtained by straight forward calculations. The divisibility condition F_w/P is achieved for $T=t_0=-\frac{p_0m}{b_0}$ and the final solution is,

$$\tilde{P}=\frac{s^2+\tilde{p}_0s}{(s+m)^2};\tilde{Q}=\frac{\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{(s+m)^2} \qquad \textbf{(11)}$$

Where

$$\tilde{p}_0=m+p_0-b_1f_0$$
$$\tilde{q}_2=q_1-f_0 \qquad \textbf{(12)}$$
$$\tilde{q}_1=q_0+q_1m-a_1f_0$$
$$\tilde{q}_0=q_0m-a_0f_0$$

And the transfer function of feedback controller is given by

$$C_b=\frac{Q_c}{P_c}=\frac{\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{s(s+\tilde{p}_0)} \qquad \textbf{(13)}$$

It is clear that (12) corresponds with the realistic PID controller

$$C_{PID}(s)=K(1+\frac{1}{T_1s}+\frac{T_Ds}{Ns+1}) \qquad \textbf{(14)}$$

III.POLE PLACEMENT USING POLYNOMIAL METHOD

Given a plant G(s), which may include the plant P(s) and feedback sensor F(s), can we find a controller C(s) that can place the roots of the characteristic polynomial is proscribed locations. This is known as the pole-placement problem.

3.1 Degree requirements

Suppose that we have a system described by the rational transfer function

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$$G(S) = \frac{B(S)}{A(S)}$$

Where

$$B(S) = b_n s^n + b_{n-1} s^{n-1} + ... + b_0$$

And

$$A(s) = a_n s^n + b_{n-1} s^{n-1} + ... + a_0$$

We ask whether there exists a controller.

$$C(s) = \frac{P(s)}{L(s)}$$

Where

$$P(s) = p_m s^m + p_{m-1} s^{m-1} + ... + p_0$$

And

$$L(s) = l_m s^m + l_{m-1} s^{m-1} + ... + l_0$$

Such that, the roots of the characteristic polynomial can be placed in arbitrary locations. Note that in this case, the "given" data is in terms of the coefficients of the plant. The degree of the controller, m, as well as the specific controller coefficients, $\{p_i\}_{i=0}^{m}$ and $\{l_i\}_{i=0}^{m}$ are to be chosen. Once we choose m, the characteristic polynomial is given by,

$$\Delta(s) = A(s)L(s) + B(s)P(s)$$

Which is an (n + m)th order polynomial. We can select (n+m) desired closed-loop pole locations, leading to a desired characteristic polynomial:

$$\Delta_d(s) = d_{n+m} s^{n+m} + d_{n+m-1} s^{n+m-1} + ... + d_0$$

and use our free parameters (the li and pi) to satisfy

$$\Delta(s) = \Delta_d(s)$$

IV. COMPARISON OF ALGEBRAIC APPROACH AND POLE PLACEMENT METHOD WITH EXAMPLES

Calculation of stabilizing PID controllers.
Example 1:

$$G_1(s) = \frac{5 - s}{(s^2 + 5s + 6)}$$

Algebraic Approach:
From eq.(7), we get

$$b_1 = 1; b_0 = 5; a_1 = 5; a_0 = 6$$

Now, For $m = 1.5$, We get,

$$\tilde{p}_0 = 1.3752$$
$$\tilde{q}_2 = 0.3752$$
$$\tilde{q}_1 = 1.2522$$
$$\tilde{q}_0 = 1.0125$$

Now the controller is

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$$C(s)=\frac{0.3752s^2+1.2522s+1.0125}{s(s+1.3752)}$$

Pole Placement Method:

$$G_1(s)=\frac{5-s}{(s^2+5s+6)}$$

Here the order of the plant is $n=2$, consider a controller of order $m=2$,

i.e. $$C(s)=\frac{p_2s^2+p_1s+p_0}{l_1s^2+l_0s}$$

With this controller the degree of the characteristic polynomial is four i.e $(n+m=4)$ which means that the desired characteristic polynomial is,

$$\Delta_d(s)=(s+3)^2(s+4)^2=s^4+10s^3+33s^2+40s+16$$

Thus, we seek parameters p1, p0, l1, and l0 such that

$$\Delta(s)=A(s)L(s)+B(s)P(s)$$
$$=l_1s^4+s^3(5l_1+l_0-p_2)+s^2(6l_1+5l_0-p_1+5p_2)+s(6l_0-p_0+5p_1)+5p_0$$

Clearly, this is possible by setting

$$l_1=1; l_0=13.5142; p_2=4.5142; p_1=23.142; p_0=28.8$$

Thus, the controller

$$C(s)=\frac{4.5142s^2+23.142s+28.8}{s^2+13.5142s}$$

SIMULATION RESULTS:

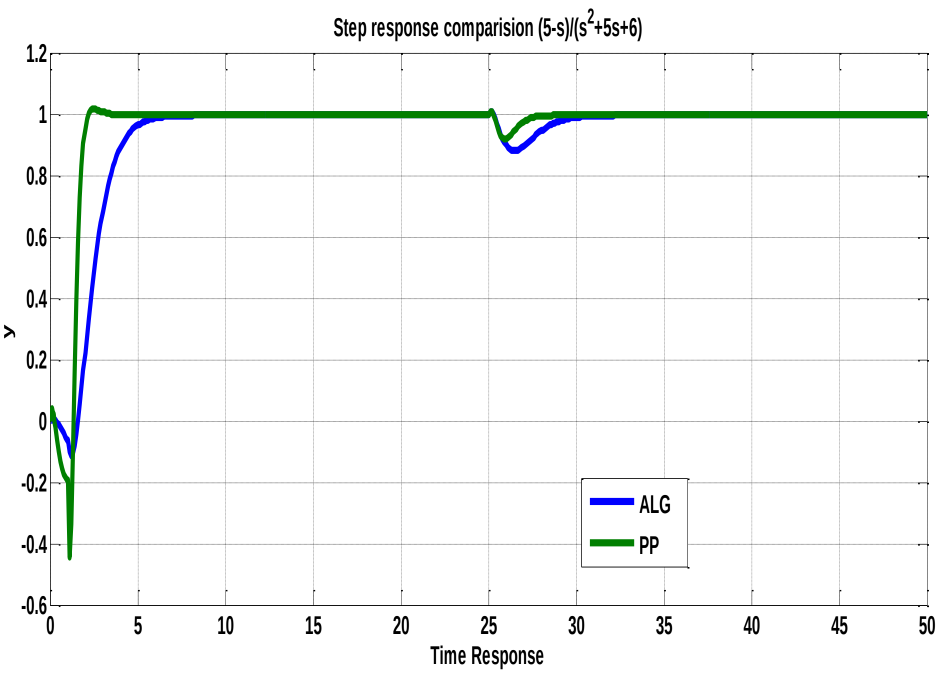

Fig.(2). Time response of G(s) with ALG and PP

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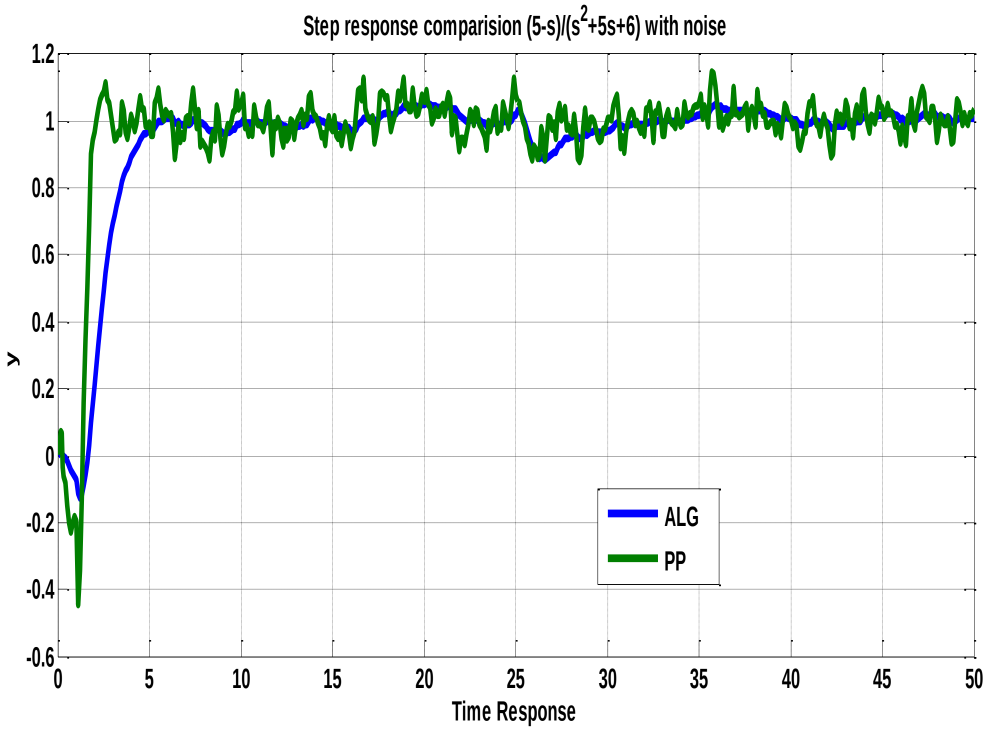
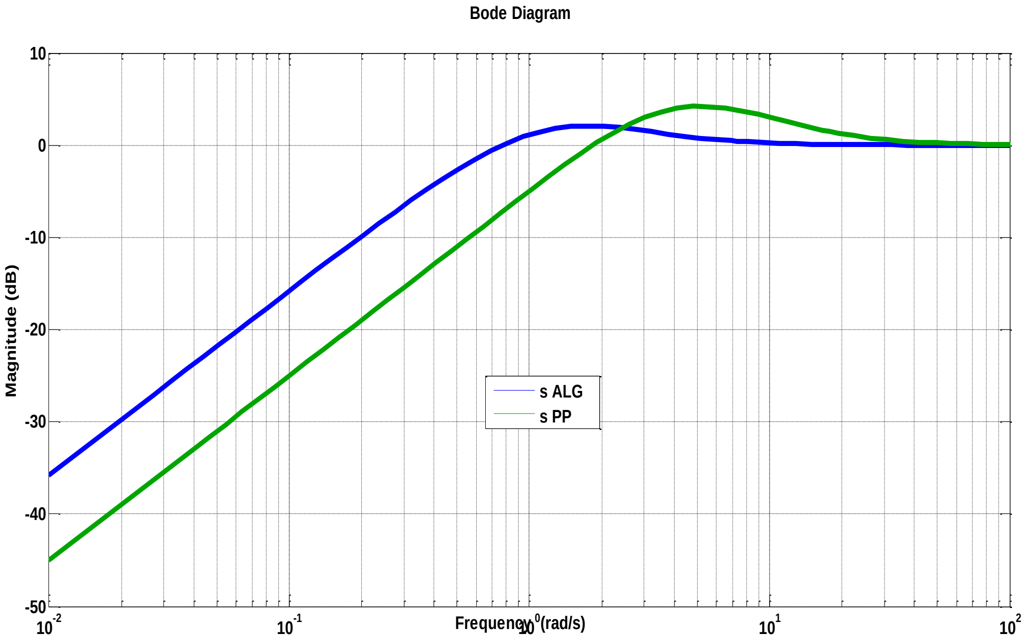


Fig.(3).Time response of G(s) under noise conditions



Fig(4). Performance requirement on sensitivity function for G(s)

Example 2:

$$G(s) = \frac{1-s}{(s^2+2s+1)}$$

Algebraic Approach

From eq. (7), we get

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$$b_1 = 1; b_0 = 1; a_1 = 2; a_0 = 1$$

Now for $m = 1$, we get,

$\tilde{p}_0 = 3$

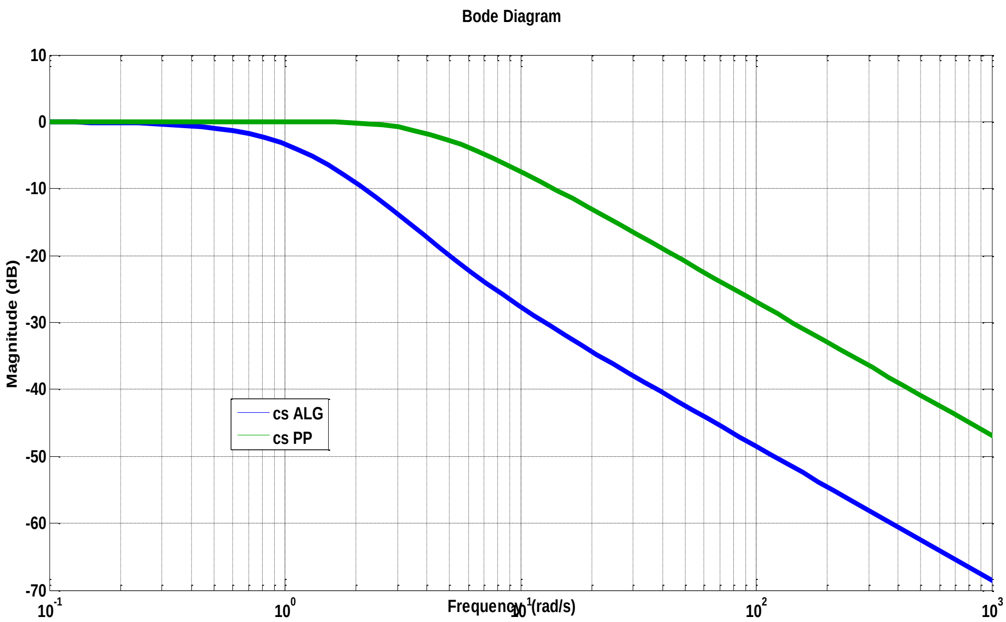
$\tilde{q}_2 = 1$

$\tilde{q}_1 = 2$

$\tilde{q}_0 = 1$

Now the controller is

$$C(s) = \frac{s^2 + 2s + 1}{s(s+3)}$$



Fig(5). Performance requirement on complimentary sensitivity function for G(s)

Pole Placement Method

$$G(s) = \frac{1-s}{(s^2 + 2s + 1)}$$

Here,

$$\Delta_d(s) = (s+1)^2(s+2)^2 = s^4 + 6s^3 + 13s^2 + 12s + 4$$

And

$$\Delta(s) = l_1 s^4 + s^3(2l_1 + l_0 - p_2) + s^2(l_1 + 2l_0 - p_1 + p_2) + s(l_0 - p_0 + p_1) + p_0$$

Thus, the controller

$$C(s) = \frac{4s^2 + 8s + 4}{s^2 + 8s}$$

Simulation result:

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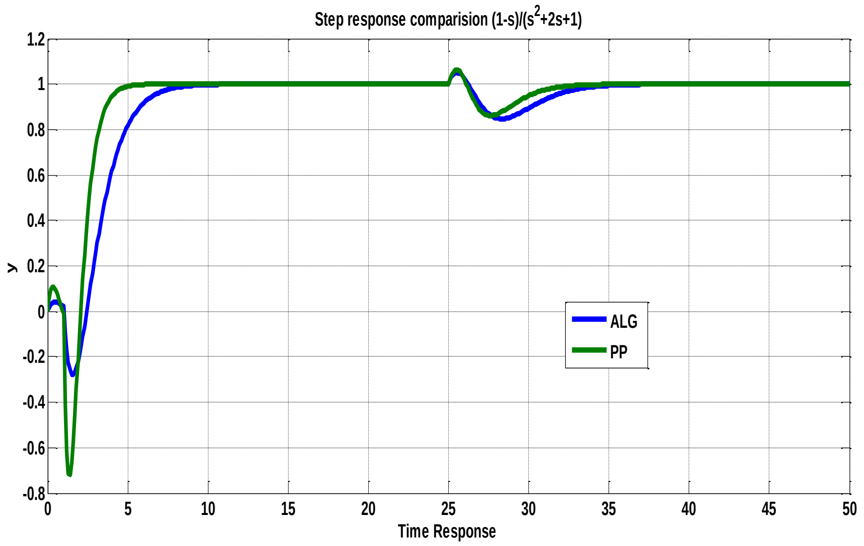


Fig.(6). Time response of G(s) with ALG and PP

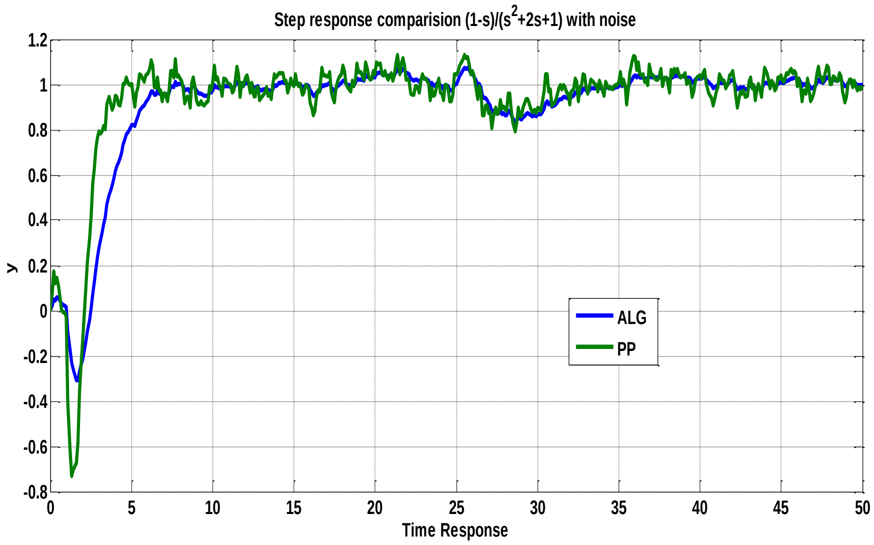
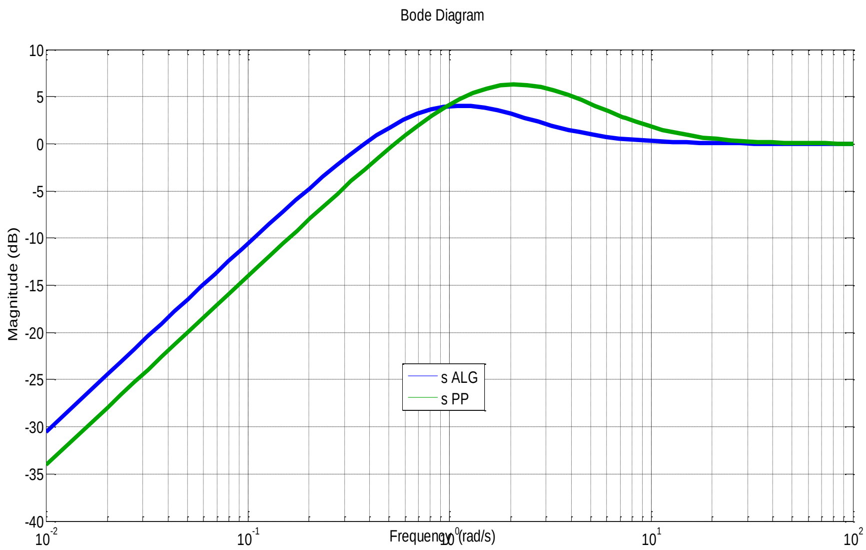


Fig.(7).Time response of G(s) under noise conditions.



Fig(8). Performance requirement on sensitivity function for G(s)

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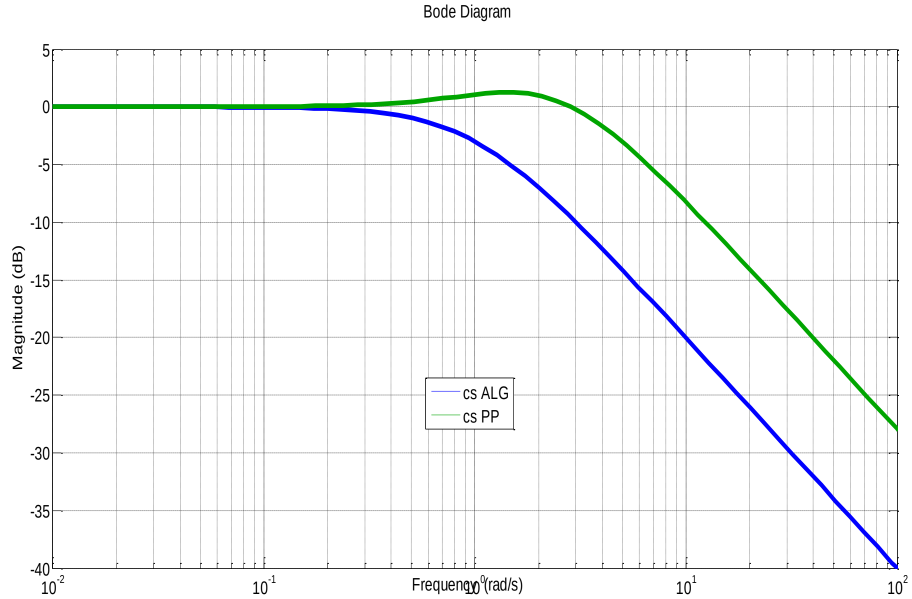

Fig(9). Performance requirement on complimentary sensitivity function for G(s)

CONCLUSIONS

These design methods were developed for SISO continuous-time non-minimum phase systems. The proposed methods enables us to tune and influence the robustness and control behaviour of the plant. Compared to the classical pole placement method for design the controller, the algebraic approach has the better control responses.

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